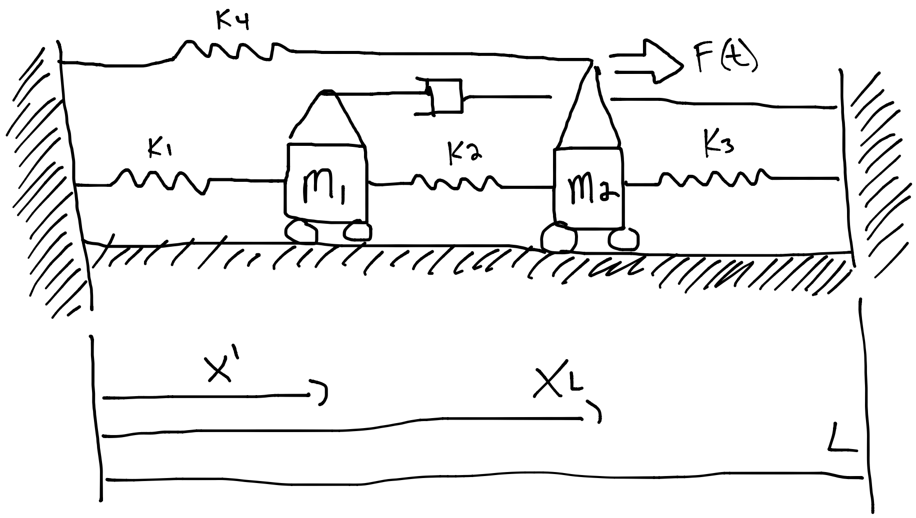
Multi-Dof → Chapter 4 Tongue

Goal: $m\ddot{\vec{x}} + c\dot{\vec{x}} + K\vec{x} = \vec{F}(t)$

Example: springs, mass, damper (mind the signs)

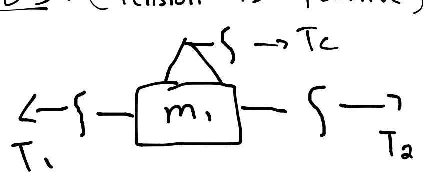


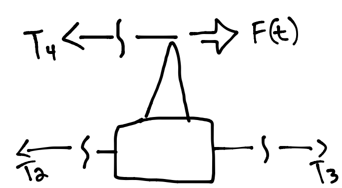
neglect width of masses

Goal: write equations   solve: a.) computer
b.) analytical

understand

FBD's: (Tension is positive)





LMB: $\sum \vec{F} = m\vec{a}$, dot with $\hat{\imath}$ $F = ma$

m_1: $T_2 + T_c - T_1 = m_1\ddot{x}_1$

*Tension is positive

m_2: $T_3 - T_2 - T_4 + F(t) = m_2\ddot{x}_2$

$T_1 = K_1(X_1 - l_1)$ $\quad l_1 =$ rest length of spring 1

$T_2 = K_2(X_2 - X_1 - l_2)$

$T_3 = K_3(L - X_2 - l_3)$

$T_4 = K_4(X_2 - l_4)$

$T_c = -c\dot{X}_1$

Equations:

m_1: $T_2 + T_c - T_1 = m_1\ddot{X}_1$

m_2: $T_3 - T_2 - T_4 + F(t) = m_2\ddot{X}_2$

Substitute all T's

$$\boxed{\begin{aligned} &m_1: K_2(X_2 - X_1 - l_2) - c\dot{X}_1 - K_1(X_1 - l_1) = m_1\ddot{X}_1 \\ &m_2: K_3(L - X_2 - l_3) - K_2(X_2 - X_1 - l_2) - K_4(X_2 - l_4) + F(t) = m_2\ddot{X}_2 \end{aligned}}$$

$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{M} \underbrace{\begin{matrix} \ddot{X}_1 \\ \ddot{X}_2 \end{matrix}}_{\ddot{\vec{X}}} + \underbrace{\begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} (K_1+K_2) & -K_2 \\ -K_2 & (K_2+K_3+K_4) \end{bmatrix}}_{K} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} -K_2 l_2 + K_1 l_1 \\ -K_3 l_3 + K_3 L + K_2 l_2 + K_4 l_4 + F(t) \end{bmatrix}}_{\vec{F}(t)}$$

$$[M]\ddot{\vec{X}} + [C]\vec{X} + [K]\vec{X} = \vec{b} + \begin{bmatrix} 0 \\ F(t) \end{bmatrix}$$

Equilibrium (solution): $F(t) = 0$, steady state

$K\vec{X} = \vec{b} \rightarrow \vec{X} = K\backslash b$ $\rightarrow$ back slash in Matlab

Define $\vec{y}$ to be $\vec{X} - \vec{X}_{ss}$

$m\ddot{\vec{y}} + c\dot{\vec{y}} + K\vec{y} = \vec{F}(t)$, and then call y, x

Simple case: no damping: $c=0$

no forcing: $F=0$

$M\ddot{\vec{x}} + K\vec{x} = \vec{0}$ ⟶ matrices are given

Method: 1. Ode 45, etc.

2. Guess: $\vec{x}(t) = \underbrace{\vec{x}}_{\text{constant vector}} e^{i\omega t}$

We find multiple solutions, add them up, and that's a solution

$$M\ddot{\vec{x}}e^{i\omega t} + K\vec{x}e^{i\omega t} = \vec{0}$$

→ need $\ddot{\vec{x}}$, $e^{i\omega t} \neq 0$

$$\boxed{\begin{aligned} \text{Solution: } -\omega^2 M\vec{x} + K\vec{x} &= \vec{0} \\ (K-\omega^2 M)\vec{x} &= \vec{0} \\ A\vec{x} &= \vec{0} \end{aligned}}$$

⟶ linear algebra problem

Matlab: >> $A\backslash\vec{0}$ ⟶ $\begin{bmatrix}0\\0\\0\end{bmatrix}$

A must be singular to have nonzero $\vec{x}$ solutions

implies: $\det(A) = 0$

implies: $\det(-\omega^2 M + K) = 0$

$\omega = \begin{bmatrix}\omega_1\\\omega_2\end{bmatrix}$ roots of "characteristic" polynomial

$$\vec{x}(t) = c_1 e^{i\omega_1 t}\vec{x}_1 + c_2 e^{i\omega_2 t}\vec{x}_2$$